Abstract

This standard specifies a Digital Signature Algorithm (DSA) which can be used to generate a digital signature. Digital signatures are used to detect unauthorized modifications to data and to authenticate the identity of the signatory. In addition, the recipient of signed data can use a digital signature in proving to a third party
that the signature was in fact generated by the signatory. This is known as nonrepudiation since the signatory cannot, at a later time, repudiate the signature.

Key words: ADP security, computer security, digital signatures, public-key cryptography, Federal Information Processing Standard.

Federal Information Processing Standards Publication 186

1994 May 19

Announcing the

DIGITAL SIGNATURE STANDARD (DSS)

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Name of Standard: Digital Signature Standard (DSS).

Category of Standard: Computer Security; Cryptography.

Explanation: This Standard specifies a Digital Signature Algorithm (DSA) appropriate for applications requiring a digital rather than written signature. The DSA digital signature is a pair of large numbers represented in a computer as strings of binary digits. The digital signature is computed using a set of rules (i.e., the DSA) and a set of parameters such that the identity of the signatory and integrity of the data can be verified. The DSA provides the capability to generate and verify signatures. Signature generation makes use of a private key to generate a digital signature. Signature verification makes use of a public key which corresponds to, but is not the same as, the private key. Each user possesses a private and public key pair. Public keys are assumed to be known to the public in general. Private keys are never shared. Anyone can verify the signature of a user by employing that user's public key. Signature generation can be performed only by the possessor of the user's private key.

A hash function is used in the signature generation process to obtain a condensed version of data, called a message digest (see Figure 1). The message digest is then input to the DSA to generate the digital signature. The digital signature is sent to the intended verifier along with the signed data (often called the message). The verifier of the message and signature verifies the signature by using the sender's public key. The same hash function must also be used in the verification process. The hash function is specified in a separate
standard, the Secure Hash Standard (SHS), FIPS 180. Similar procedures may be used to generate and verify signatures for stored as well as transmitted data.

Approving Authority: Secretary of Commerce.


Applicability: This standard is applicable to all Federal departments and agencies for the protection of unclassified information that is not subject to section 2315 of Title 10, United States Code, or section 3502(2) of Title 44, United States Code. This standard shall be used in designing and implementing public-key based signature systems which Federal departments and agencies operate or which are operated for them under contract. Adoption and use of this standard is available to private and commercial organizations.

Applications: The DSA authenticates the integrity of the signed data and the identity of the signatory. The DSA may also be used in proving to a third party that data was actually signed by the generator of the signature. The DSA is intended for use in electronic mail, electronic funds transfer, electronic data interchange, software distribution, data storage, and other applications which require data integrity assurance and data origin authentication.

Implementations: The DSA may be implemented in software, firmware, hardware, or any combination thereof. NIST is developing a validation program to test implementations for conformance to this standard. Information about the planned validation program can be obtained from the National Institute of Standards and Technology, Computer Systems Laboratory, Attn: DSS Validation, Gaithersburg, MD 20899.

Export Control: Implementations of this standard are subject to Federal Government export controls as specified in Title 15, Code of Federal Regulations, Parts 768 through 799. Exporters are advised to contact the Department of Commerce, Bureau of Export Administration for more information.

Patents: The Department of Commerce is not aware of any patents that would be infringed by this standard.

Implementation Schedule: This standard becomes effective December 1, 1994.


Cross Index:

a. Federal Information Resources Management Regulations (FIRMR) subpart
Qualifications: The security of a digital signature system is dependent on maintaining the secrecy of users' private keys. Users must therefore guard against the unauthorized acquisition of their private keys. While it is the intent of this standard to specify general security requirements for generating digital signatures, conformance to this standard does not assure that a particular implementation is secure. The responsible authority in each agency or department shall assure that an overall implementation provides an acceptable level of security. This standard will be reviewed every five years in order to assess its adequacy.

Waiver Procedure: Under certain exceptional circumstances, the heads of Federal departments and agencies may approve waivers to Federal Information Processing Standards (FIPS). The head of such agency may redelegate such authority only to a senior official designated pursuant to section 3506(b) of Title 44, United States Code. Waiver shall be granted only when:

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b. Compliance with a standard would cause a major adverse financial impact on the operator which is not offset by Government-wide savings.

Agency heads may act upon a written waiver request containing the information detailed above. Agency heads may also act without a written waiver request when they determine that conditions for meeting the standard cannot be met. Agency heads may approve waivers only by a written decision which explains the basis on which the agency head made with required finding(s). A copy of each decision, with procurement sensitive or classified portions clearly identified, shall be sent to: National Institute of Standards and Technology; ATTN: FIPS Waiver Decisions, Technology Building, Room B-154, Gaithersburg, MD 20899.

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When the determination on a waiver applies to the procurement of equipment and/or services, a notice of the waiver determination must be published in the Commerce Business Daily as a part of the notice of solicitation for offers of an
acquisition or, if the waiver determination is made after that notice is published, by amendment to such notice.

A copy of the waiver, any supporting documents, the document approving the waiver and any accompanying documents, with such deletions as the agency is authorized and decides to make under 5 United States Code Section 552(b), shall be part of the procurement documentation and retained by the agency.

Where to Obtain Copies of the Standard: Copies of this publication are for sale by the National Technical Information Service, U.S. Department of Commerce, Springfield, VA 22161. When ordering, refer to Federal Information Processing Standards Publication 186 (FIPSPUB186), and identify the title. When microfiche is desired, this should be specified. Prices are published by NTIS in current catalogs and other issuances. Payment may be made by check, money order, deposit account or charged to a credit card accepted by NTIS.

Federal Information Processing Standards Publication XX

1994 May 19

Specifications for the DIGITAL SIGNATURE STANDARD (DSS)

1. INTRODUCTION

This publication prescribes the Digital Signature Algorithm (DSA) for digital signature generation and verification. Additional information is provided in Appendices 1 through 5.

2. GENERAL

When a message is received, the recipient may desire to verify that the message has not been altered in transit. Furthermore, the recipient may wish to be certain of the originator’s identity. Both of these services can be provided by the DSA. A digital signature is an electronic analogue of a written signature in that the digital signature can be used in proving to the recipient or a third party that the message was, in fact, signed by the originator. Digital signatures may also be generated for stored data and programs so that the integrity of the data and programs may be verified at any later time.

This publication prescribes the DSA for digital signature generation and verification. In addition, the criteria for the public and private keys required by the algorithm are provided.

3. USE OF THE DSA ALGORITHM

The DSA is used by a signatory to generate a digital signature on data and by a verifier to verify the authenticity of the signature. Each signatory has a public and
private key. The private key is used in the signature generation process and the public key is used in the signature verification process. For both signature generation and verification, the data which is referred to as a message, M, is reduced by means of the Secure Hash Algorithm (SHA) specified in FIPS YY. An adversary, who does not know the private key of the signatory, cannot generate the correct signature of the signatory. In other words, signatures cannot be forged. However, by using the signatory's public key, anyone can verify a correctly signed message. A means of associating public and private key pairs to the corresponding users is required. That is, there must be a binding of a user's identity and the user's public key. This binding may be certified by a mutually trusted party. For example, a certifying authority could sign credentials containing a user's public key and identity to form a certificate. Systems for certifying credentials and distributing certificates are beyond the scope of this standard. NIST intends to publish separate document(s) on certifying credentials and distributing certificates.

4. DSA PARAMETERS

The DSA makes use of the following parameters:

1. p = a prime modulus, where 2L-1 < p < 2L for 512 <= L <= 1024 and L a multiple of 64
2. q = a prime divisor of p - 1, where 2159 < q < 2160
3. g = h(p-1)/q mod p, where h is any integer with 1 < h < p - 1 such that h(p-1)/q mod p > 1 (g has order q mod p)
4. x = a randomly or pseudorandomly generated integer with 0 < x < q
5. y = gx mod p
6. k = a randomly or pseudorandomly generated integer with 0 < k < q

The integers p, q, and g can be public and can be common to a group of users. A user's private and public keys are x and y, respectively. They are normally fixed for a period of time. Parameters x and k are used for signature generation only, and must be kept secret. Parameter k must be regenerated for each signature.

Parameters p and q shall be generated as specified in Appendix 2, or using other FIPS approved security methods. Parameters x and k shall be generated as specified in Appendix 3, or using other FIPS approved security methods.

5. SIGNATURE GENERATION

The signature of a message M is the pair of numbers r and s computed according to the equations below:

r = (gk mod p) mod q and

s = (k-1(SHA(M) + xr)) mod q.
In the above, k-1 is the multiplicative inverse of k, mod q; i.e., \((k - 1) k \mod q = 1\) and \(0 < k - 1 < q\). The value of SHA(M) is a 160-bit string output by the Secure Hash Algorithm specified in FIPS 180. For use in computing s, this string must be converted to an integer. The conversion rule is given in Appendix 2.2.

As an option, one may wish to check if \(r = 0\) or \(s = 0\). If either \(r = 0\) or \(s = 0\), a new value of k should be generated and the signature should be recalculated (it is extremely unlikely that \(r = 0\) or \(s = 0\) if signatures are generated properly).

The signature is transmitted along with the message to the verifier.

6. SIGNATURE VERIFICATION

Prior to verifying the signature in a signed message, p, q and g plus the sender’s public key and identity are made available to the verifier in an authenticated manner.

Let \(M\), \(r\), and \(s\) be the received versions of \(M\), \(r\), and \(s\), respectively, and let \(y\) be the public key of the signatory. To verify the signature, the verifier first checks to see that \(0 < r < q\) and \(0 < s < q\); if either condition is violated the signature shall be rejected. If these two conditions are satisfied, the verifier computes

\[
w = (s) - 1 \mod q
\]

\[
u_1 = ((SHA(M))w) \mod q
\]

\[
u_2 = ((r)w) \mod q \quad v = (((g)u_1 (y)u_2) \mod p) \mod q.
\]

If \(v = r\), then the signature is verified and the verifier can have high confidence that the received message was sent by the party holding the secret key \(x\) corresponding to \(y\). For a proof that \(v = r\) when \(M = M\), \(r = r\), and \(s = s\), see Appendix 1.

If \(v\) does not equal \(r\), then the message may have been modified, the message may have been incorrectly signed by the signatory, or the message may have been signed by an impostor. The message should be considered invalid.

APPENDIX 1. A PROOF THAT \(v = r\)

This appendix is for informational purposes only and is not required to meet the standard.

The purpose of this appendix is to show that if \(M = M\), \(r = r\) and \(s = s\) in the signature verification then \(v = r\). We need the following easy result.
LEMMA. Let p and q be primes so that q divides p - 1, h a positive integer less than p, and \( g = h(p-1)/q \mod p \). Then \( gq \mod p = 1 \), and if \( m \mod q = n \mod q \), then \( gm \mod p = gn \mod p \).

Proof: We have

\[
gq \mod p = \left( h(p-1)/q \mod p \right) q \mod p = h(p-1) \mod p = 1 \]

by Fermat's Little Theorem. Now let \( m \mod q = n \mod q \), i.e., \( m = n + kq \) for some integer \( k \). Then

\[
gm \mod p = gn + kq \mod p = (gn gkq) \mod p = ((gn \mod p) (gq \mod p)k) \mod p = gn \mod p \]

since \( gq \mod p = 1 \).

We are now ready to prove the main result.

THEOREM. If \( M^+ = M \), \( r^+ = r \), and \( s^+ = s \) in the signature verification, then \( v = r^+ \).

Proof: We have

\[
w = (s^+)^{-1} \mod q = s^{-1} \mod q
\]

\[
u_1 = ((SHA(M^+))w) \mod q = ((SHA(M))w) \mod q
\]

\[
u_2 = ((r^+)w) \mod q = (rw) \mod q.
\]

Now \( y = gx \mod p \), so that by the lemma,

\[
v = ((gu_1 yu_2) \mod p) \mod q = ((SHA(M)w yrw) \mod p) \mod q
\]

\[
= ((SHA(M)w gxrw) \mod p) \mod q = ((SHA(M)w + xr)w) \mod p) \mod q.
\]

Also

\[
s = (k^{-1}(SHA(M) + xr)) \mod q.
\]

Hence
\[ w = (k(SHA(M) + xr) - 1) \mod q \]

\[(SHA(M) + xr)w \mod q = k \mod q.\]

Thus by the lemma,

\[ v = (gk \mod p) \mod q \]

= r

= rb \cdot p

**APPENDIX 2. GENERATION OF PRIMES FOR THE DSA**

This appendix includes algorithms for generating the primes p and q used in the DSA. These algorithms require a random number generator (see Appendix 3), and an efficient modular exponentiation algorithm. Generation of p and q shall be performed as specified in this appendix, or using other FIPS approved security methods.

**2.1. A PROBABILISTIC PRIMALITY TEST**

In order to generate the primes p and q, a primality test is required.

There are several fast probabilistic algorithms available. The following algorithm is a simplified version of a procedure due to M.O. Rabin, based in part on ideas of Gary L. Miller. [See Knuth, The Art of Computer Programming, Vol. 2, Addison-Wesley, 1981, Algorithm P, page 379.] If this algorithm is iterated n times, it will produce a false prime with probability no greater than 1/4n. Therefore, n \( \geq 50 \) will give an acceptable probability of error. To test whether an integer is prime:

**Step 1.** Set i = 1 and n \( \geq 50 \).

**Step 2.** Set w = the integer to be tested, w = 1 + 2am, where m is odd and 2a is the largest power of 2 dividing w - 1.

**Step 3.** Generate a random integer b in the range 1 < b < w.

**Step 4.** Set j = 0 and z = bm mod w.

**Step 5.** If j = 0 and z = 1, or if z = w - 1, go to step 9.

**Step 6.** If j > 0 and z = 1, go to step 8.

**Step 7.** j = j + 1. If j < a, set z = z2 mod w and go to step 5.
Step 8. w is not prime. Stop.

Step 9. If i < n, set i = i + 1 and go to step 3. Otherwise, w is probably prime.

2.2. GENERATION OF PRIMES

The DSS requires two primes, p and q, satisfying the following three conditions:

a. $2^{159} < q < 2^{160}$
b. $2^{L-1} < p < 2^{L}$ for a specified L, where $L = 512 + 64j$ for some $0 \leq j \leq 8$
c. q divides p - 1.

This prime generation scheme starts by using the SHA and a user supplied SEED to construct a prime, q, in the range $2^{159} < q < 2^{160}$. Once this is accomplished, the same SEED value is used to construct an X in the range $2^{L-1} < X < 2^{L}$. The prime, p, is then formed by rounding X to a number congruent to 1 mod $2q$ as described below.

An integer x in the range $0 \leq x < 2^g$ may be converted to a g-long sequence of bits by using its binary expansion as shown below:

$$x = x_1*2^{g-1} + x_2*2^{g-2} + ... + x_{g-1}*2 + x_g \rightarrow \{ x_1, ..., x_g \}.$$ 

Conversely, a g-long sequence of bits $\{ x_1, ..., x_g \}$ is converted to an integer by the rule

$$\{ x_1, ..., x_g \} \rightarrow x_1*2^{g-1} + x_2*2^{g-2} + ... + x_{g-1}*2 + x_g.$$ 

Note that the first bit of a sequence corresponds to the most significant bit of the corresponding integer and the last bit to the least significant bit.

Let $L - 1 = n*160 + b$, where both b and n are integers and $0 \leq b < 160$.

Step 1. Choose an arbitrary sequence of at least 160 bits and call it SEED. Let g be the length of SEED in bits.

Step 2. Compute

$$U = SHA[SEED] \ XOR \ SHA[(SEED+1) \ mod \ 2^g].$$

Step 3. Form q from U by setting the most significant bit (the 2159 bit) and the least significant bit to 1. In terms of boolean operations, $q = U \ OR \ 2159 \ OR \ 1$. Note that $2^{159} < q < 2^{160}$.

Step 4. Use a robust primality testing algorithm to test whether q is prime.
Step 5. If q is not prime, go to step 1.

Step 6. Let counter = 0 and offset = 2.

Step 7. For k = 0,...,n let

\[ V_k = SHA[(SEED + offset + k) mod 2^g]. \]

1A robust primality test is one where the probability of a non-prime number passing the test is at most 2^-80.

Step 8. Let W be the integer

\[ W = V_0 + V_1*2^{160} + ... + V_{n-1}*2^{(n-1)*160} + (V_n \mod 2^b) * 2^{n*160} \]

and let X = W + 2^{L-1}. Note that 0 ≤ W < 2^{L-1} and hence 2^{L-1} ≤ X < 2^L.

Step 9. Let c = X \mod 2^q and set p = X - (c - 1). Note that p is congruent to 1 \mod 2^q.

Step 10. If p < 2^{L-1}, then go to step 13.


Step 12. If p passes the test performed in step 11, go to step 15.

Step 13. Let counter = counter + 1 and offset = offset + n + 1.

Step 14. If counter ≥ 2^{12} = 4096 go to step 1, otherwise (i.e. if counter < 4096) go to step 7.

Step 15. Save the value of SEED and the value of counter for use in certifying the proper generation of p and q.

APPENDIX 3. RANDOM NUMBER GENERATION FOR THE DSA

Any implementation of the DSA requires the ability to generate random or pseudorandom integers. Such numbers are used to derive a user’s private key, x, and a user’s per message secret number, k. These randomly or pseudorandomly generated integers are selected to be between 0 and the 160-bit prime q (as specified in the standard). They shall be generated by the techniques given in this appendix, or using other FIPS approved security methods. One FIPS approved pseudorandom integer generator is supplied in Appendix C of ANSI X9.17, "Financial Institution Key Management (Wholesale)."
Other pseudorandom integer generators are given in this appendix. These permit generation of pseudorandom values of x and k for use in the DSA. The algorithm in section 3.1 may be used to generate values for x. An algorithm for k and r is given in section 3.2. The latter algorithm allows most of the signature computation to be precomputed without knowledge of the message to be signed.

The algorithms employ a one-way function G(t,c), where t is 160 bits, c is b bits (160 ≤ b ≤ 512) and G(t,c) is 160 bits. One way to construct G is via the Secure Hash Algorithm (SHA), as defined in the Secure Hash Standard (SHS). The 160-bit message digest output of the SHA algorithm when message M is input is denoted by SHA(M). A second method for constructing G is to use the Data Encryption Standard (DES). The construction of G by these techniques is discussed in sections 3.3 and 3.4 of this appendix.

In the algorithms in sections 3.1 and 3.2, a secret b-bit seed-key is used. The algorithm in section 3.1 optionally allows the use of a user provided input. If G is constructed via the SHA as defined in section 3.3, then b is between 160 and 512. If DES is used to construct G as defined in section 3.4, then b is equal to 160.

3.1. ALGORITHM FOR COMPUTING m VALUES OF x

Let x be the signer's private key. The following may be used to generate m values of x:

Step 1. Choose a new, secret value for the seed-key, XKEY.

Step 2. In hexadecimal notation let

\[ t = 67452301 \text{ EFCDAB89} \text{ 98BADCFE} \text{ 10325476} \text{ C3D2E1F0}. \]

This is the initial value for H0 || H1 || H2 || H3 || H4 in the SHS.

Step 3. For j = 0 to m - 1 do

- a. XSEEDj = optional user input.
- b. XVAL = (XKEY + XSEEDj) mod 2b.
- c. xj = G(t,XVAL) mod q.
- d. XKEY = (1 + XKEY + xj) mod 2b.

3.2. ALGORITHM FOR PRECOMPUTING ONE OR MORE k AND r VALUES

This algorithm can be used to precompute k, k-1, and r for m messages at a time. Algorithm:

Step 1. Choose a secret initial value for the seed-key, KKEY.
Step 2. In hexadecimal notation let

t = EFCDAB89 98BADCFE 10325476 C3D2E1F0 67452301.

This is a cyclic shift of the initial value for H0 || H1 || H2 || H3 || H4 in the SHS.

Step 3. For j = 0 to m - 1 do

a. k = G(t,KKEY) mod q.
b. Compute kj-1 = k-1 mod q.
c. Compute rj = (gk mod p) mod q.
d. KKEY = (1 + KKEY + k) mod 2b.

Step 4. Suppose M0 , ... , Mm-1 are the next m messages. For j = 0 to m - 1 do

a. Let h = SHA(Mj).
b. Let sj = (kj-1(h + xrj)) mod q.

Step 3 permits precomputation of the quantities needed to sign the next m messages. Step 4 can begin whenever the first of these m messages is ready. The execution of step 4 can be suspended whenever the next of the m messages is not ready. As soon as steps 4 and 5 have completed, step 3 can be executed, and the results saved until the first member of the next group of m messages is ready.

In addition to space for KKEY, two arrays of length m are needed to store r0 , ... , rm-1 and k0-1 , ... , km-1-1 when they are computed in step 3. Storage for s0 , ... , sm-1 is only needed if the signatures for a group of messages are stored; otherwise sj in step 4 can be replaced by s and a single space allocated.

3.3. CONSTRUCTING THE FUNCTION G FROM THE SHA

G(t,c) may be constructed using steps (a) - (e) in section 7 of the Specifications for the Secure Hash Standard. Before executing these steps, {Hj} and M1 must be initialized as follows:

i. Initialize the {Hj} by dividing the 160 bit value t into five 32-bit segments as follows:

t = t0 || t1 || t2 || t3 || t4

Then Hj = tj for j = 0 through 4.

ii. There will be only one message block, M1, which is initialized as follows:

M1 = c || 0512-b

(The first b bits of M1 contain c, and the remaining (512-b) bits are set to zero).
Then steps (a) through (e) of section 7 are executed, and \( G(t,c) \) is the 160 bit string represented by the five words:

\[ H_0 \ || \ H_1 \ || \ H_2 \ || \ H_3 \ || \ H_4 \] at the end of step (e).

### 3.4. CONSTRUCTING THE FUNCTION \( G \) FROM THE DES

Let \( a \) XOR \( b \) denote the bitwise exclusive-or of bit strings \( a \) and \( b \). Suppose \( a_1, a_2, b_1, b_2 \) are 32-bit strings. Let \( b_1' \) be the 24 least significant bits of \( b_1 \). Let \( K = b_1' \ || \ b_2 \) and \( A = a_1 \ || \ a_2 \). Define

\[ \text{DES}_{b_1,b_2}(a_1,a_2) = \text{DES}(A) \]

In the above, \( \text{DES}(A) \) represents ordinary DES encryption of the 64-bit block \( A \) using the 56-bit key \( K \). Now suppose \( t \) and \( c \) are each 160 bits. To compute \( G(t,c) \):

**Step 1.** Write

\[ t = t_1 \ || \ t_2 \ || \ t_3 \ || \ t_4 \ || \ t_5 \]

\[ c = c_1 \ || \ c_2 \ || \ c_3 \ || \ c_4 \ || \ c_5 \]

In the above, each \( t_i \) and \( c_i \) is 32 bits.

**Step 2.** For \( i = 1 \) to 5 do

\[ x_i = t_i \ \text{XOR} \ c_i \]

**Step 3.** For \( i = 1 \) to 5 do

\[ b_1 = c((i+3) \mod 5) + 1 \]

\[ b_2 = c((i+2) \mod 5) + 1 \]

\[ a_1 = x_i \]

\[ a_2 = x(i \mod 5) + 1 \ \text{XOR} \ x((i+3) \mod 5) + 1 \]

\[ y_{i,1} \ || \ y_{i,2} = \text{DES}_{b_1,b_2}(a_1,a_2) \ (y_{i,1}, y_{i,2} = 32 \text{ bits}) \]

**Step 4.** For \( i = 1 \) to 5 do

\[ z_i = y_{i,1} \ \text{XOR} \ y((i+1) \mod 5)+1,2 \ \text{XOR} \ y((i+2) \mod 5)+1,1 \]

**Step 5.** Let

\[ G(t,c) = z_1 \ || \ z_2 \ || \ z_3 \ || \ z_4 \ || \ z_5 \]
APPENDIX 4. GENERATION OF OTHER QUANTITIES

This appendix is for informational purposes only and is not required to meet the standard.

The algorithms given in this appendix may be used to generate the quantities g, k-1, and s-1 used in the DSS. To generate g:

Step 1. Generate p and q as specified in Appendix 2.

Step 2. Let e = (p - 1)/q.

Step 3. Set h = any integer, where 1 < h < p - 1 and h differs from any value previously tried.

Step 4. Set g = he mod p.

Step 5. If g = 1, go to step 3. To compute the multiplicative inverse n-1 mod q for n with 0 < n < q, where 0 < n-1 < q:

Step 1. Set i = q, h = n, v = 0, and d = 1.

Step 2. Let t = i DIV h, where DIV is defined as integer division.

Step 3. Set x = h.

Step 4. Set h = i - tx.

Step 5. Set i = x.

Step 6. Set x = d.

Step 7. Set d = v - tx.

Step 8. Set v = x.

Step 9. If h > 0, go to step 2.

Step 10. Let n-1 = v mod q.

Note that in step 10, v may be negative. The v mod q operation should yield a value between 1 and q - 1 inclusive.

APPENDIX 5. EXAMPLE OF THE DSA
This appendix is for informational purposes only and is not required to meet the standard.

Let \( L = 512 \) (size of \( p \)). The values in this example are expressed in hexadecimal notation. The \( p \) and \( q \) given here were generated by the prime generation standard described in appendix 2 using the 160-bit SEED:

\[
d5014e4b\ 60ef2ba8\ b6211b40\ 62ba3224\ e0427dbd
\]

With this SEED, the algorithm found \( p \) and \( q \) when the counter was at 38.

\( x \) was generated by the algorithm described in appendix 3, section 3.1, using the SHA to construct \( G \) (as in appendix 3, section 3.3) and a 160-bit XSEED:

\[
\text{XSEED =}
\]

\[
\text{bd029bbe\ 7f51960b\ cf9eddb2b\ 61f06f0f\ eb5a38b6}
\]

\[
\text{t =}
\]

\[
67452301\ \text{EFCDAB89\ 98BADCFE\ 10325476\ C3D2E1F0}
\]

\[
x = G(t,\text{XSEED}) \mod q
\]

\( k \) was generated by the algorithm described in appendix 3, section 3.2, using the SHA to construct \( G \) (as in appendix 3, section 3.3) and a 160-bit KSEED:

\[
\text{KSEED =}
\]

\[
687a66d9\ 0648f993\ 867e121f\ 4ddf9dddb\ 01205584
\]

\[
\text{t =}
\]

\[
\text{EFCDAB89\ 98BADCFE\ 10325476\ C3D2E1F0\ 67452301}
\]

\[
k = G(t,\text{KSEED}) \mod q
\]

Finally:

\[
h = 2
\]

\[
p =
\]

\[
d411a4a0\ e393f6aa\ b0f08b14\ d1845866\ 5b3e4dbd\ ce254454\ 3fe365cf\ 71c86224\ 12db6e7d\ d02bbe13\ d88c58d7\ 263e9023\ 6af17ac8\ a9fe5f2a\ 9cc81f42\ 7fc543f7
\]

\[
q =
\]
\[ b_{20db0b1} \ 01df0c66 \ 24fc1392 \ ba55f77d \ 577481e5 \]

\[ g = \]
\[ b_{3085510} \ 021f9990 \ 49a9e7cd \ 3872ce99 \ 58186b50 \ 07e7adaf \ 25248b58 \ a3dc4f71 \ 781d21f2 \ df89b717 \ 47bd54b3 \ 23bbecc4 \ 43ec1d3e \ 020dadab \ bf782257 \ 8255c104 \]

\[ x = \]
\[ 6b2cd935 \ d0192d54 \ e2c942b5 \ 74c80102 \ c8f8ef67 \]

\[ k = \]
\[ 79577ddc \ aafddc03 \ 8b865b19 \ f8eb1ada \ 8a2838c6 \]

\[ k_{inv} = \]
\[ 2784e3d6 \ 72d972a7 \ 4e22c67f \ 4f4f726e \ cc751efa \]

\[ M = \text{ASCII form of "abc" (See FIPS PUB YY, Appendix A)} \]

\[ \text{SHA}(M) = \]
\[ 0164b8a9 \ 14cd2a5e \ 74c4f7ff \ 082c4d97 \ f8df880 \]

\[ y = \]
\[ b_{32fbec0} \ 3175791d \ f08c3f86 \ 1c81df7d \ e7e0cba7 \ f1c4f726 \ 9bb12d6c \ 628784fb \ 742e66ed \ 315754df \ e38b5984 \ e94d3725 \ 37f655cb \ 3ea4767c \ 878cbd2d \ 783ee662 \]

\[ r = \]
\[ 9b77f705 \ 4c81531c \ 4e46a469 \ 2fbfe0f7 \ 7f7ebff2 \]

\[ s = \]
\[ 95b4f608 \ 1f8f890e \ 4b5a199e \ f10ffe21 \ f52b2d68 \]

\[ w = \]
\[ 0ceb5f6b \ 875f6b67 \ 7e093134 \ df70b0d4 \ 3226680c \]

\[ u1 = \]
\[ 347089a2 \ 9897273b \ fc7a774f \ a70e0e0e \ 153bcc95 \]
\[ u_2 = 793d9312 \text{a41b88af aa2c1bd9 49ec3bee 2e75d2f5} \]

\[ g_{u1 \mod p} = 57a198ab \text{2c8ea0b6 4810767a ff732fb2 da5fcafb 278889f1 96b60b9c 1285b848} \]
\[ 1d08505e \text{201a5c68 523a15ee 2fb62a56 d141dc4d 71925ef0 6acde0a5 b89c5671} \]

\[ y_{u2 \mod p} = 5d983d20 \text{be604e23 fb19bec8 7860490a 41b865dc 0f5623f4 0724a795} \]
\[ 021bcd8c 93a39ddf 51cae380 fb6d682a 676608f7 65227ff0 5e44ccf4 9767e4a6 0832d33f \]

\[ v = 9b77f705 \text{4c81531c 4e46a469 2fbfe0f7 7f7ebff2} \]

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